"You Do It" Solution

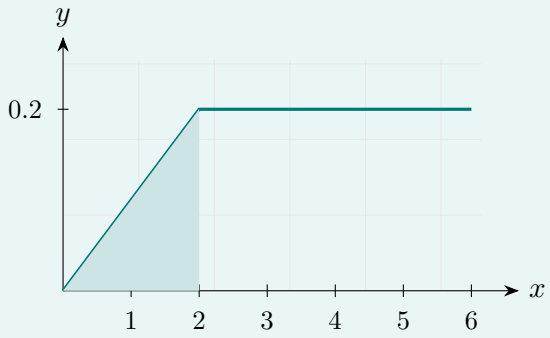
Lecture 7A: The Normal Distribution and Other Continuous Distribution

Prepared as a classroom-ready detailed solution set for ECOM5005

Exercise 1: Probability from a Density Curve

Question

Consider the density curve below. Find the probability that x is less than 2.



Detailed Solution

Step 1: Translate the probability statement into an area.
For a continuous random variable, a probability is represented by the **area under the density curve**. Therefore,

$$P(X < 2) = \text{area under the density curve to the left of } x = 2.$$

From the graph, the density begins at $x = 0$, so the required area is the region from $x = 0$ to $x = 2$.
Step 2: Identify the shape of the required region.
Between $x = 0$ and $x = 2$, the density rises linearly from $(0, 0)$ to $(2, 0.2)$. The area below this portion of the curve is therefore a **triangle** with

$$\text{base} = 2 - 0 = 2, \qquad \text{height} = 0.2.$$

Step 3: Calculate the triangular area.
Using the area formula

$$\text{Area of a triangle} = \frac{1}{2}(\text{base})(\text{height}),$$

we obtain

$$P(X < 2) = \frac{1}{2}(2)(0.2) = 0.20.$$

Hence,

$$\boxed{P(X < 2) = 0.20}.$$

Equivalently,

$$\boxed{P(X < 2) = 20\%}.$$

Exercise 1: Interpretation and Final Answer

Probability Is Area Under a Density Curve

The central idea is that the vertical height of a continuous density curve is a **density**, not a probability by itself. To calculate a probability over an interval, we find the **area under the curve** over that interval.
Thus, although the graph has height 0.2 at $x = 2$, the answer 0.20 comes from the triangular area to the left of 2, not simply from reading the height of the curve.

Geometric Calculation

The required region is the triangle from $x = 0$ to $x = 2$:

$$P(X < 2) = \frac{1}{2} \times \underbrace{2}_{\text{base}} \times \underbrace{0.2}_{\text{height}} = 0.20.$$

Therefore, **20% of the total probability area lies below** $x = 2$.

Check that the Density Curve Is Valid

The triangular area from 0 to 2 is 0.20. From $x = 2$ to $x = 6$, the curve forms a rectangle of height 0.2 and width 4:

$$(6 - 2)(0.2) = 0.80.$$

Hence the total area is

$$0.20 + 0.80 = 1.00,$$

which confirms that the displayed curve is a valid probability density function.

Answer Summary

Final answer:

$$\boxed{P(X < 2) = 0.20 = 20\%.}$$

Reason: The required probability is the area of the triangular region under the density curve between $x = 0$ and $x = 2$.

Final Check

For continuous probability distributions, remember:

- probability is **area under the density curve**;
- the total area under a valid density curve must equal 1;
- the probability of any exact single value is 0, so

$$P(X < 2) = P(X \leq 2).$$